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Robust Flight Controller Design That Takes into Account Handling Quality

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A robust flight controller design method, which satisfies handling quality requirements, is shown. The expanded control anticipation parameter is introduced as a new handling quality criterion to treat high-order dynamics. The relations to the C^* criterion and equivalent systems are shown. The proposed criterion is formulated as a linear matrix inequality, and a robust flight controller that satisfies handling-quality requirements can be designed numerically. In addition, a cost index with a time-weighting function that evaluates several time response features is introduced into controller design. An example that illustrates flight controller design of Multi-Purpose Aviation Laboratory airplane is presented, and flight testing is performed.

Nomenclature

$A > 0$	=	A is positive definite.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	=	$C(sI - A)^{-1}B + D$
$e^{\Gamma t}$	=	$I + \Gamma t + (1/2!)\Gamma^2 t^2 + (1/3!)\Gamma^3 t^3 + \cdots$
l_p	=	distance of the pilot's station ahead of the c.g.
q	=	pitch rate
q_g	=	gust gradient component along the pitch axis
r	=	pilot stick input
U_0	=	equilibrium velocity along the X axis
u	=	velocity change along the X axis
u_g	=	gust component along the X axis
V_{co}	=	crossover velocity(ratio in C^* between pitch rate and normal acceleration change at pilot's station)
w	=	velocity change along the Z axis
w_g	=	gust component along the Z axis
α	=	angle of attack
Δn	=	normal acceleration change at the c.g.
Δn_p	=	normal acceleration change at pilot's station
δ_{DLC}	=	direct-lift-control (DLC) angle
δ_{DLCc}	=	DLC command
δ_e	=	elevator angle
δ_{ec}	=	elevator deflection command
δ_t	=	power lever deflection
δ_{tc}	=	power lever command
θ	=	pitch attitude

Introduction

ONE of the most important factors of a flight-control system is handling quality. Several criteria for handling quality have been established, and a controller must be designed to meet these requirements. In Ref. 1 the handling-quality criterion of longitudinal dynamics was developed, which can be equivalently formulated as eigenvalue requirements.[2] This criterion is based on short-period dynamics. The C^* criterion is the requirement in the time domain to step input and has been applied to several fly-by-wire airplanes. The dynamics of recent airplane have become complex, and so high-order dynamics is essential in evaluating handling quality. Reference 3 states that pure time delay in airplane dynamics degrades the pilot rating. Phase lag at high frequency acts as time delay and so must be taken into consideration when evaluating handling quality. To treat high-order systems, an equivalent system was proposed.[4] This method is based on the approximation of a high-order system as a low-order equivalent system and equivalent time delay.

A flight-control system must be designed to satisfy handling quality requirements. Generally, the present flight-control system depends on the trial-and-error design method using a simple controller. Modern control theory treats mainly robust stability and so is inadequate for satisfying handling-quality requirements. However modern control theory methods, such as the H_∞ controller obtained via the linear matrix-inequality (LMI) approach, have the significant advantages of robust stability, simplicity of design procedure, and multi-objective design.[5] If a requirement is formulated in LMIs, it can be combined with other useful LMI conditions, for example, robust stability and pole assignment. Although some applications of the H_∞ controller to flight-control systems have been developed (for example, see Refs. 6 and 7), a method by which to satisfy handling-quality requirements has not yet been reported. The problem lies in formulating handling quality as a useful design condition, and the formulated condition must adequately represent the handling quality of complex airplane dynamics.

Eigenstructure assignment[8,9] is a design method that can meet the control-anticipation-parameter (CAP) requirements. To satisfy robust stability in addition to handling quality, the robust eigenstructure assignment method has been proposed.[10–12] Although these methods seem to be adequate for a flight-control system, more procedures are necessary for practical use. CAP requirements are based on the short-period mode and so effective only if other modes do not affect time responses. In addition, pilots must consider many time response features, such as damping, settling time, and overshoot. The eigenstructure assignment cannot easily satisfy CAP requirements of high-order dynamics and these time response features. For a recently designed airplane that has advanced flight-control systems containing numerous complex dynamics, the eigenstructure approach is insufficient.

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In this paper, a new handling-quality criterion is proposed. The criterion is shown to be a simple expansion of CAP and is related to equivalent systems. In addition, a cost index of time response, by which the time response feature can be shaped, is introduced. The criterion and cost index are used for controller design. It must be emphasized that these constraints are formulated using whole dynamics, which includes controller dynamics and actuator dynamics. In addition, because the constraints are expressed in LMI form, multiobjective design is possible. This method is combined with a static H_∞ controller and then applied to Multi-Purpose Aviation Laboratory airplane (MuPAL-α) of the Japan Aerospace Exploration Agency. Flight testing using the designed controller was performed.

Design Requirements

The longitudinal dynamics of an airplane are considered, and design requirements are established: 1) good time response to pilot stick input, 2) decrease in pitch-rate response to gust input, and 3) robust stability subject to multiplicative uncertainty on the input side.

To meet these requirements, a two-block controller, which consists of a feed-forward block and a feedback block, is used (Fig. 1). State-space expressions of the airplane dynamics and controller are given as follows.

Airplane P:

$$\dot{\boldsymbol{x}}_P = A_P\boldsymbol{x}_P + B_P u_P \tag{1a}$$

$$y_P = C_P\boldsymbol{x}_P \tag{1b}$$

Feed-forward controller K:

$$\dot{x}_K = A_K x_K + B_K r \tag{2a}$$

$$u_K = C_K\boldsymbol{x}_K + D_K r \tag{2b}$$

Feedback controller F:

$$\dot{\boldsymbol{x}}_F = A_F\boldsymbol{x}_F + B_F y_P \tag{3a}$$

$$u_F = C_F\boldsymbol{x}_F + D_F y_P \tag{3b}$$

where $\boldsymbol{x}_P$ is the open-loop state vector of the airplane that contains q and r is δ_{ec}. From Eqs. (1–3) and $u_P = u_K + u_F$, the closed-loop system becomes as follows:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Br \tag{4a}$$

$$q = C_q\boldsymbol{x} \tag{4b}$$

$$\boldsymbol{x} = \begin{bmatrix} x_P^T & x_F^T & x_K^T \end{bmatrix}^T$$

$$A = \begin{bmatrix} A_P + B_P D_F C_P & B_P C_F & B_P C_K \\ B_F C_P & A_F & 0 \\ 0 & 0 & A_K \end{bmatrix}, \qquad B = \begin{bmatrix} B_P D_K \\ 0 \\ B_K \end{bmatrix}$$

where C_q is a vector, that extracts q from the extended state vector x. The controller is designed by formulating design requirements (1–3) in LMIs with respect to a closed-loop system.

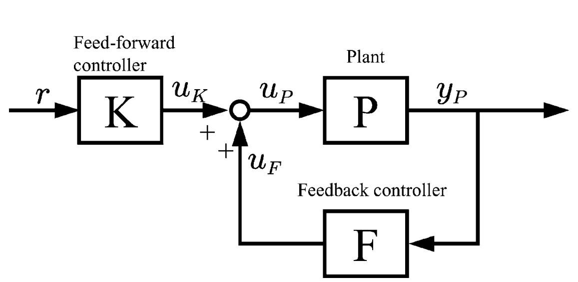

Fig. 1 Closed-loop.

Handling Quality of Longitudinal Dynamics

CAP and C^* Criterion

First, the control anticipation parameter (CAP) is discussed. CAP is defined based on the assumption that a pilot initiates a maneuver predicting steady-state normal acceleration with initial pitch acceleration:

$$\text{CAP} = \frac{\ddot{\theta}/\delta_{\text{ec}}|_{t=0}}{\Delta n/\delta_{\text{ec}}|_{t=\infty}} \tag{5}$$

The requirements for CAP were formulated in LMIs, and the controller design method is shown in Ref. 13.

In addition, the C^* criterion is also successful, and constrains the step response of $C^* = \Delta n_p + (V_{\text{co}}/g)q$ in the envelope characterized by overshoot, rise rate, and settling time (Fig. 2). The relation between these flying qualities is easily shown. For the open-loop system and $U_0 \ll V_{\text{co}}$,

$$C^* \big/ C^*_{\text{ss}} \simeq \text{CAP} \times U_0/g \times t \tag{6}$$

is satisfied. CAP represents the rise rate of C^*. But, in the system that contains other dynamics such as phugoid dynamics in addition to short-period mode, $\Delta n/\delta_{\text{ec}}|_{t=\infty} = 0$ is satisfied and CAP cannot be defined appropriately in Eq. (5). CAP is defined assuming that the pilot notices the response immediately after the stick input when the short-period mode is dominant. Usually, Eq. (5) is evaluated using the short-period parameter. If other dynamics such as phugoid, controller, or actuator dynamics strongly affect the time responses, a criterion other than CAP is necessary. Equation (6) assumes that the criterion, which defines some feature of C^*, is advantageous.

CAP* Criterion

Based on the preceding argument, a new criterion is proposed.

CAP*:

$$\text{CAP}^* = \frac{g}{2U_0}\frac{\ddot{\theta}/\delta_{\text{ec}}|_{t=0}}{q_p/\delta_{\text{ec}}} \tag{7}$$

where q_p is the maximum value of pitch rate immediately after step input. If $U_0 \ll V_{\text{co}}$, then

$$C^* \big/ C^*_p \simeq \text{CAP}^* \times 2U_0/g \times t \tag{8}$$

is satisfied. The assumption $U_0 \ll V_{\text{co}}$ is used only to show the relation with time response [Eqs. (6) and (8)]. Any other theoretical development doesnot depend on the assumption. C^*_p is the maximum value of C^* immediately after step input. From Eq. (8) and Fig. 2, CAP* is a criterion that describes the rise rate of C^* like CAP [see Eq. (6)]. These relations indicate that CAP* is a reasonable expansion of CAP. Here, Eq. (7) is rewritten in another form.

Theorem 1: Substitution using Eqs. (4) and (7) for pilot step stick input becomes

$$\text{CAP}^* \simeq -C_q AB/C_q B \cdot g/U_0 \tag{9}$$

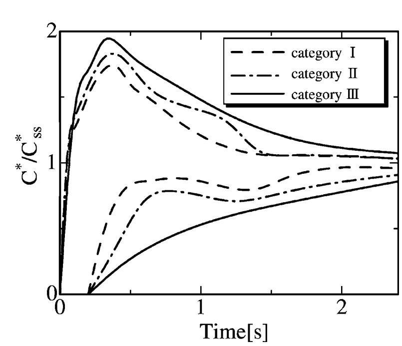

Fig. 2 C^* criterion.

Proof: The step response of Eq. (4) is

$$\boldsymbol{x} = e^{At}A^{-1}B - A^{-1}B, \qquad q = C_q\boldsymbol{x} \tag{10}$$

$$q = C_qBt + \tfrac{1}{2}C_qABt^2 + \mathcal{O}(t^3)$$

$$= \tfrac{1}{2}C_qAB(t + C_qB/C_qAB)^2 - (C_qB)^2\big/2C_qAB + \mathcal{O}(t^3)$$

follows. Then,

$$q_p \simeq -(C_qB)^2\big/2C_qAB$$

is obtained. From the preceding equation and

$$\dot{q}(0) = C_qB \tag{11}$$

Eq. (9) is derived. □

From Ref. 13,

$$\mathrm{CAP} = -\frac{C_qB}{C_qA^{-1}B}\frac{g}{U_0}$$

is obtained. If Eq. (9) is satisfied with accuracy,

$$\frac{\mathrm{CAP}}{\mathrm{CAP}^*} \simeq \frac{(C_qB)^2}{(C_qAB)(C_qA^{-1}B)}$$

$$\frac{|C_qB|^2\sigma_{\min}(A)}{|C_q|^2|B|^2\sigma_{\max}(A)} \le \frac{\mathrm{CAP}}{\mathrm{CAP}^*} \le \frac{|C_qB|^2\sigma_{\max}(A)}{|C_q|^2|B|^2\sigma_{\min}(A)}$$

are obtained. There is a strong relation between CAP and CAP*. In addition, it is shown in the next section that CAP* evaluates phase lag at high frequency.

CAP* and Equivalent Systems

Advanced airplanes and flight-control systems have led to complicated high-order airplane dynamics. To evaluate the handling quality of the high-order airplane dynamics, equivalent systems have been developed.[4] This method is based on the approximation of high-order dynamics to low-order equivalent systems and equivalent time delay. The requirements for short-period mode[2] are imposed on the low-order equivalent system. However time delay in airplane dynamics degrades handling quality,[3] and equivalent time delay is important in order to estimate pilot rating.[4] Next, it is shown that CAP* can evaluate both the handling quality of equivalent systems and phase lag at high frequency.

Let the whole airplane dynamics from elevator command to pitch rate be $P(s)$. $P(s)$ is assumed to be approximated as follows:

$$P(s) \simeq P_L(s)P_H(s) \triangleq \tilde{P}(s)$$

where $P_L(s)$ is a low-order approximated system that represents low-frequency dynamics and $P_H(s)$ represents high-frequency dynamics. Let CAP* of $\tilde{P}(s)$ and $P_L(s)$ be $\tilde{\mathrm{CAP}}^*$ and CAP^*_L, respectively. State-space equations of $P_L(s)$ and $P_H(s)$ are given as follows:

$$P_L = \left[\begin{array}{c|c} A_L & B_L \\ \hline C_L & 0 \end{array}\right], \qquad P_H = \left[\begin{array}{c|c} A_H & B_H \\ \hline C_H & D_H \end{array}\right]$$

And the series connection becomes

$$\tilde{P} = \left[\begin{array}{cc|c} A_L & B_LC_H & B_LD_H \\ 0 & A_H & B_H \\ \hline C_L & 0 & 0 \end{array}\right]$$

$$\triangleq \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & 0 \end{array}\right]$$

Substituting the preceding equation into Eq. (9) gives

$$\tilde{\mathrm{CAP}}^* \simeq -\tilde{C}\tilde{A}\tilde{B}/\tilde{C}\tilde{B}\cdot g/U_0$$

$$= \mathrm{CAP}^*_L - C_HB_H/D_H\cdot g/U_0 \tag{12}$$

Equation (12) means that the CAP* of the whole dynamics is CAP* of the low-order model affected by high-frequency dynamics. The approximation of Eq. (12) derives from Eq. (9). Let the peak time of pitch-rate response be t_p; the approximation error of Eq. (12) becomes $\mathcal{O}(t_p^3)$.

Using a simplified model, the effects of high-frequency dynamics becomes clear. It is assumed that $P_H(s)$ is approximated by a first-order model as follows:

$$P_H(s) = K\times(T_2s+1)/(T_1s+1), \qquad A_H = -1/T_1, \qquad B_H = 1$$

$$C_H = K\left(-T_2\big/T_1^2 + 1/T_1\right), \qquad D_H = KT_2/T_1$$

From

$$\tan[\angle P_H(j\omega)] = \frac{\mathrm{Im}[P_H(j\omega)]}{\mathrm{Re}[P_H(j\omega)]}$$

$$= \frac{(T_2 - T_1)\omega}{1 + T_1T_2\omega^2}$$

$$\int_{\sqrt{(\omega_0-1)/T_1T_2}}^{\sqrt{(100\omega_0-1)/T_1T_2}} \tan[\angle P_H(j\omega)]\,\mathrm{d}\omega = -\frac{C_HB_H}{D_H} \tag{13}$$

is obtained. This equation is satisfied for all $\omega_0 > 1$. Equation (13) evaluates the phase lag of high-frequency dynamics. If the phase lag at high frequency is large, the response becomes sluggish, and CAP and CAP* of a low-order system cannot evaluate response correctly. Equations (12) and (13) reveal that CAP* of the whole system evaluated the rise rate of step response appropriately. It can be concluded that CAP* is a good measure of sensitivity of step response and does not conflict with the equivalent systems method.

Quadratic Cost Index with Time-Weighting Function

In Ref. 2, the damping ratio is constrained as one of the handling-quality requirements, with the intention of guaranteeing appropriate response rate and overshoot with respect to pilot input. However damping ratio provides an imperfect estimation of the response of the closed-loop system of Eq. (4). To adjust the response of Eq. (4) to step input, the following index is introduced:

$$J = \int_0^\infty \boldsymbol{\Delta x}_n^T e^{\Gamma^T t} W e^{\Gamma t}\boldsymbol{\Delta x}_n\,\mathrm{d}t, \qquad \boldsymbol{\Delta x}_n = \frac{x(t) - \boldsymbol{x}(\infty)}{|\boldsymbol{x}(\infty)|} \tag{14}$$

where $W > 0$ and Γ is a matrix that satisfies $A\Gamma = \Gamma A$.

If Γ is a stable matrix, $e^{\Gamma t}$ converges. $[x(t) - x(\infty)]/|x(\infty)|$ is normalized error, and the proposed index is a time-weighted error function. For the appropriate Γ, the rise rate, overshoot, and settling time can be estimated.

H_∞ Control Problem

Design requirements (2) and (3) are satisfied by the H_∞ controller. Let a gust disturbance be d_g, the controlled output for gust d_g be $z_g = q$, the disturbance from the multiplicative uncertainty on the input side be d_U, and the controlled output be z_U. Introducing these disturbances and outputs, Eq. (4) becomes

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B_1\boldsymbol{d} + B_2r \tag{15a}$$

$$\boldsymbol{z} = C_1\boldsymbol{x} + D_{11}\boldsymbol{d} + D_{12}r \tag{15b}$$

where

$$\boldsymbol{z} = \left[z_g^T \quad z_U^T\right]^T, \qquad \boldsymbol{d} = \left[d_g^T \quad d_U^T\right]^T$$

Let the transfer function from d_g to z_g be T_{dzg} and that from d_U to z_U be T_{dzU}. Design requirements (2) and (3) can be satisfied with decreasing $\|T_{\mathrm{dzg}}\|_\infty$ and $\|T_{\mathrm{dzU}}\|_\infty$. This is formulated as a mixed sensitivity problem.

Design requirements (2), (3):

$$\|T_{\mathrm{dz}}\|_\infty < \gamma$$

where T_{dz} is a transfer function from d to z.

Design Method

The design method via LMIs that meets design requirements (1–3) is shown. All design requirements are formulated as LMIs, and the controller is obtained numerically.

Using the next theorem, the CAP* requirement is formulated as LMIs.

Theorem 2: Let $\beta_1 < \beta_2$, $\epsilon > 0$,

$$\hat{\beta}_1 = U_0\beta_1/g, \qquad \hat{\beta}_2 = U_0\beta_2/g$$

$$\mu = (\hat{\beta}_1 + \hat{\beta}_2)/2, \qquad \eta = \tfrac{1}{4}[\epsilon(\hat{\beta}_2 - \hat{\beta}_1)]^2 \tag{16}$$

If there exists P such that

$$\begin{bmatrix} \eta & C_q(\mu I + A)P \\ P(\mu I + A)^T C_q^T & P \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} P & B \\ B^T & I \end{bmatrix} > 0 \tag{18}$$

$$\epsilon^2 + C_q B + (C_q B)^T + I < 0 \tag{19}$$

then CAP* satisfies the following inequality:

$$\beta_1 < \mathrm{CAP}^* < \beta_2 \tag{20}$$

Proof: Applying the Schur complement to Eq. (17), and from Eq. (18),

$$|C_q(\mu I + A)B| < \sqrt{\eta} \tag{21}$$

is obtained. From

$$(C_q B + I)(C_q B + I)^T > 0$$

and Eq. (19),

$$|C_q B| > \epsilon \tag{22}$$

is satisfied. From Eqs. (21) and (22), Eq. (20) is derived. □

Formulating the index of Eq. (14) as LMIs, several time response features can be considered in controller design.

Theorem 3: Let $\xi > 0$. If there exists S such that

$$\begin{bmatrix} -[S(A+\Gamma)^T + (A+\Gamma)S] & S \\ S & W^{-1} \end{bmatrix} > 0 \tag{23}$$

$$S > \xi^{-1} I \tag{24}$$

then $J < \xi$.

Proof: Applying the Schur complement to Eq. (23),

$$(A+\Gamma)^T T + T(A+\Gamma) < -W, \qquad T = S^{-1} \tag{25}$$

is obtained. Because $T > 0$, $A + \Gamma$ is stable matrix. From Eqs. (10) and (25),

$$J = \frac{1}{|A^{-1}B|^2} \times \int_0^\infty (A^{-1}B)^T e^{(A+\Gamma)^T t} W e^{(A+\Gamma)t} A^{-1}B \,\mathrm{d}t$$

$$< \frac{1}{|A^{-1}B|^2}(A^{-1}B)^T T A^{-1} B < \lambda_{\max}(T) \tag{26}$$

follows. Then, from Eq. (24), $J < \xi$ is obtained. □

Using the bounded real lemma, the H_∞ control problem can be written as LMIs.

Theorem 4: $\|T_{\mathrm{dz}}\| < \gamma$, if and only if there exists U such that

$$\begin{bmatrix} AU + UA^T & UC_1^T & B_1 \\ C_1 U & -\gamma I & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{bmatrix} > 0, \qquad U > 0$$

Here, static feed-forward controller and static state feedback controller are considered. From $u_K = Kr$, $u_F = Fx_P$, $u_P = u_F + u_K$, and Eq. (15), the generalized plant is obtained:

$$\dot{x} = (A + B_2 F)x + B_1 d + B_2 K r \tag{27a}$$

$$z = (C_1 + D_{12}F)x + D_{11}d \tag{27b}$$

$$q = C_q x \tag{27c}$$

Using this plant, design requirements are formulated as a convex problem.

Theorem 5: Let $\beta_1 < \beta_2$, $\epsilon > 0$, Γ such that $A\Gamma = \Gamma A$, $\xi > 0$, and $\gamma > 0$, and μ and η are solutions of Eq. (16). If there exist P, K, and X such that

$$\begin{bmatrix} \eta & C_q(\mu I + A)P + C_q B_2 X \\ P(\mu I + A)^T C_q^T + X^T B_2^T C_q^T & P \end{bmatrix} > 0 \tag{28a}$$

$$\begin{bmatrix} P & B_2 K \\ (B_2 K)^T & I \end{bmatrix} > 0 \tag{28b}$$

$$\epsilon^2 + C_q B_2 K + (C_q B_2 K)^T + I < 0 \tag{28c}$$

$$\begin{bmatrix} -\left[P(A+\Gamma)^T + (A+\Gamma)P + X^T B_2^T + B_2 X\right] & P \\ P & W^{-1} \end{bmatrix} > 0 \tag{28d}$$

$$P > \xi^{-1} I \tag{28e}$$

$$\begin{bmatrix} AP + PA^T + B_2 X + X^T B_2^T & PC_1^T + X^T D_{12}^T & B_1 \\ C_1 P + D_{12} X & -\gamma I & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{bmatrix} > 0 \tag{28f}$$

then, Eq. (27) satisfies

$$\beta_1 < CAP^* < \beta_2, \qquad J < \xi, \qquad \|T_{\mathrm{dz}}\|_\infty < \gamma$$

Feedback gain is $F = XP^{-1}$.

Proof: Equation (28) is obtained applying the common solution $P = S = U$ and parameter transformation $X = FP$ to Theorems 2–4. □

If a strong relation is assumed between CAP and CAP*, then the design parameters β_1 and β_2 are determined as follows. Let the requirements for CAP be $\alpha_1 < \mathrm{CAP} < \alpha_2$. For the open-loop system, let $\mathrm{CAP} = \mathrm{CAP}_0$ and $\mathrm{CAP}^* = \mathrm{CAP}_0^*$. Then $\beta_1 = \alpha_1 \mathrm{CAP}_0^*/\mathrm{CAP}_0$ and $\beta_2 = \alpha_2 \mathrm{CAP}_0^*/\mathrm{CAP}_0$. Here, ϵ is a free parameter even if CAP and CAP* are fixed, and from Eqs. (11) and (22), ϵ can be interpreted to be a parameter that constrains gear ratio, which must be determined based on comments from test pilots. Γ and ξ are determined by referring to the C^* response and based on test pilot comments. Here, γ is a robust stability parameter, which must be minimized under the preceding handling quality constraints.

Design procedure:

1) β_1 and β_2 are determined from CAP constraints.
2) ϵ is determined, which constrains the elevator gear ratio.
3) μ and η are calculated from Eq. (16).
4) Γ and ξ are determined from time response requirements.
5) Equation (28) is solved.

Design Example

Controller Design

The design method is applied to MuPAL-α (Ref. 14; Fig. 3) flying at a height of 5000 ft with $U_0 = 66.5$ m/s. MuPAL-α has three control

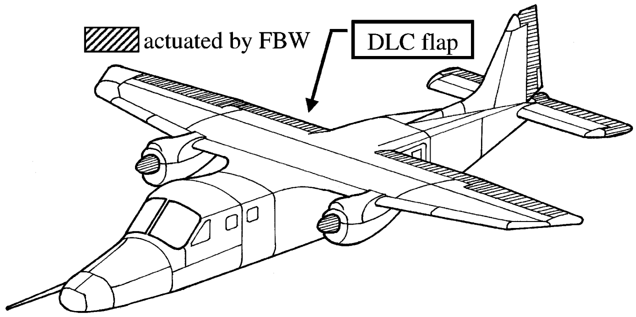

Fig. 3 MuPAL-α.

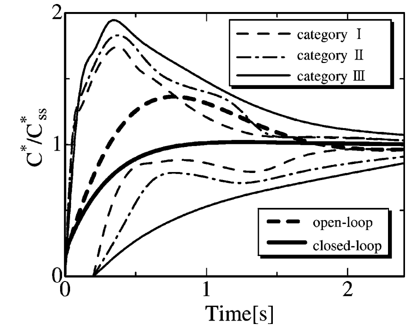

Fig. 4 C^* response.

inputs of longitudinal dynamics: elevator angle δ_e, direct-lift-control (DLC) angle δ_{DLC}, and power lever deflection δ_t.

The airframe dynamics are modeled using a four-degree-of-freedom model. The elevator and DLC actuators are modeled by constant gain, and the engine is modeled by first-order lag. The matrices in Eq. (27) become

$$A = \begin{bmatrix} -0.016 & 0.172 & -6.090 & -9.764 & 0.013 \\ -0.183 & -1.093 & 64.546 & -0.915 & -0.003 \\ 0.008 & -0.069 & -1.903 & 0.007 & 0.000 \\ 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.217 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.016 & 0.172 & 0.078 & 0.326 & 0.336 & 0 \\ -0.183 & -1.093 & -0.828 & -3.457 & -6.103 & 0 \\ 0.008 & -0.069 & -0.943 & -2.978 & 0.760 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.196 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.326 & 0.336 & 0 \\ -3.457 & -6.103 & 0 \\ -2.978 & 0.760 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3.196 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{11} = 0_{4\times 6}, \qquad D_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = [u(\text{m/s}) \quad w(\text{m/s}) \quad q(\text{rad/s}) \quad \theta(\text{rad}) \quad \delta_t(\%)]^T$$

$$u_P = [\delta_{\text{ec}} \quad \delta_{\text{DLC}c} \quad \delta_{\text{tc}}]^T$$

where δ_{ec} is the elevator command, $\delta_{\text{DLC}c}$ is the DLC command, and δ_{tc} is the power lever command.

As design requirement (1), the category A level 1 requirement of $0.28 < \text{CAP} < 3.6^2$ and the C^* criterion is imposed. From test pilot comments, overshoot of C^* decreased. As design requirements (2) and (3), $\|T_{\text{dz}}\|_\infty$ is minimized.

In an open-loop system, $CAP_0 = 0.9577$. Then, let $\beta_1 = 0.3\text{CAP}_0^*$ and $\beta_2 = 3.7\text{CAP}_0^*$. Let vector in B_2 according to elevator input be B_{δ_e} and let $\epsilon = 0.6|C_q B_{\delta_e}|$. In addition, let $\Gamma = -1.0 \times I$, $W = 0.001 \times I$, and $\xi = 2.6$. These parameters are adjusted according to [design procedure] with slight trial and error.

The controller that satisfies Eq. (28) becomes

$$K = \begin{bmatrix} 0.578 \\ -0.494 \\ 0.138 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.002 & -0.009 & 0.557 & 0.330 & 0.000 \\ -0.000 & 0.011 & -0.169 & -0.250 & -0.000 \\ -0.000 & 0.000 & -0.001 & -0.000 & -0.002 \end{bmatrix}$$

The designed controller attains $\|T_{\text{dz}}\|_\infty = 4.9775$. In an open-loop system, $\|T_{\text{dz}}\|_\infty = 5.1554$. In particular, the H_∞ norm of the transfer function from gust d_g to pitch rate q is 1.4770 in an open-loop system and 0.2251 in a closed-loop system. The controller thus has increased robustness with respect to gust disturbance. In the short-period mode of a closed-loop system, CAP = 0.8215, which satisfies the CAP requirement. Figure 4 shows C^* responses of open-/closed-loop system, where $l_p = 5$ m and $V_{\text{co}} = 99.8$ m/s. Although the open-loop system does not satisfy the C^* criterion, the controller improves the response. The closed-loop system meets the requirement. The eigenvalues of the open-loop system are $-1.4968 \pm 2.0841i$, $-0.0091 \pm 0.1779i$, -2.2171, and those of the closed-loop system are $-2.3083 \pm 1.1537i$, $-0.1098 \pm 0.1661i$, -2.2236. The feedback controller increases the damping. The short-period damping ratio requirement of category A level 1 is $0.35 < \zeta_s < 1.30^2$. The short-period damping ratio of the closed-loop system is $\zeta_s = 0.9558$, which meets the requirement.

Figure 5 shows the responses to doublet stick input, where the outputs from controllers F and K are discretized with zero-order hold with 50 Hz. The handling qualities seem not to be changed significantly, except for the slight decrease in the response peak. If the pilot feels that the responses are sluggish, the responses can be improved by designing a new controller with a larger ϵ. Figure 6 shows the gust responses. The gust parameters are as follows:

$$[t < \pi/\omega_{\text{nscl}}]$$

$$u_g = 0, \qquad w_g = (V_m/2)\{1 - \cos(\omega_{\text{nscl}} t)\}$$

$$q_g = -(V_m \omega_{\text{nscl}}/2)\sin(\omega_{\text{nscl}} t)$$

$$[t \geq \pi/\omega_{\text{nscl}}]$$

$$u_g = 0, \qquad w_g = V_m, \qquad q_g = 0$$

where ω_{nscl} is the closed-loop natural frequency $d_g = [u_g \quad w_g \quad q_g]^T$ and $V_m = 4.5$ m/s (equivalent airspeed) = 4.3 m/s (true airspeed).

Because the frequency of gust is equal to the short-period mode frequency of the closed-loop system, the response becomes large. However compared to the open-loop system, the responses of the closed-loop system are decreased by the controller.

In actual flight, robust stability to uncertainty in actuator dynamics is important. The controller ensures the stability subject to multiplicative uncertainty from the input side. When the actuator uncertainty consists of both gain change and time lag, the stability margin for the uncertainty is calculated analytically using small gain theorem. Although the analysis is conservative, stability is guaranteed

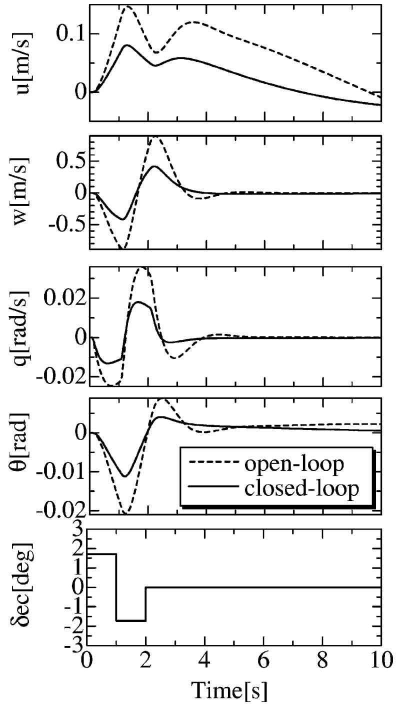

Fig. 5 Doublet responses.

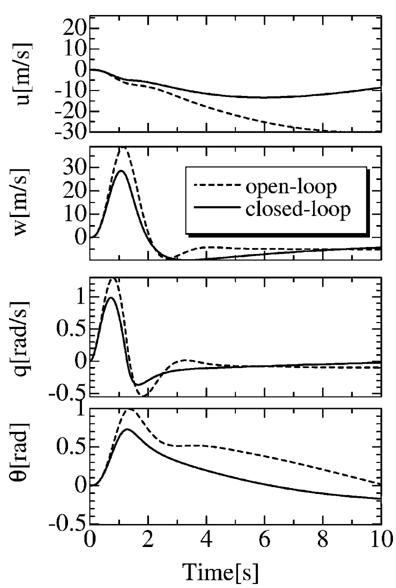

Fig. 6 Gust responses.

for a gain change of 0–1.57 and a time lag change of −0.45–0.45 s. Figure 7 shows the response to the same gust with elevator, DLC, and engine dynamics gain changes, multiplied by 0.7, and a time lag increase of 0.3 s. Although slight oscillation occurs, the stability is maintained.

Flight test

Next, the designed controller was validated by flight testing. The designed-controller-equipped MuPAL-α was flown under the design conditions. The responses to doublet input of open and closed-loop system are shown in Fig. 8. Although more careful flight tests and test pilot comments are necessary in order to validate the design method, the designed controller was confirmed to operate well. In particular, the robust stability was confirmed and the overshoot of the closed-loop system decreases as a result of design requirements.

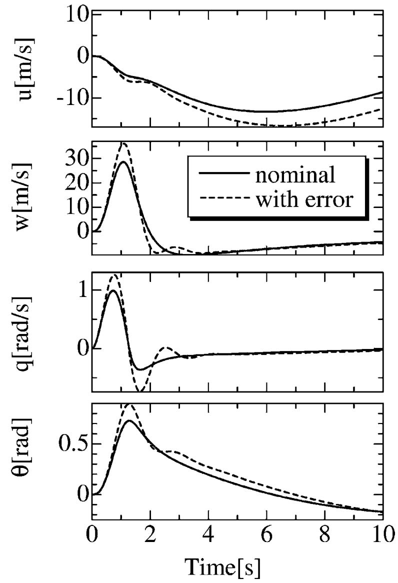

Fig. 7 Modeling errors.

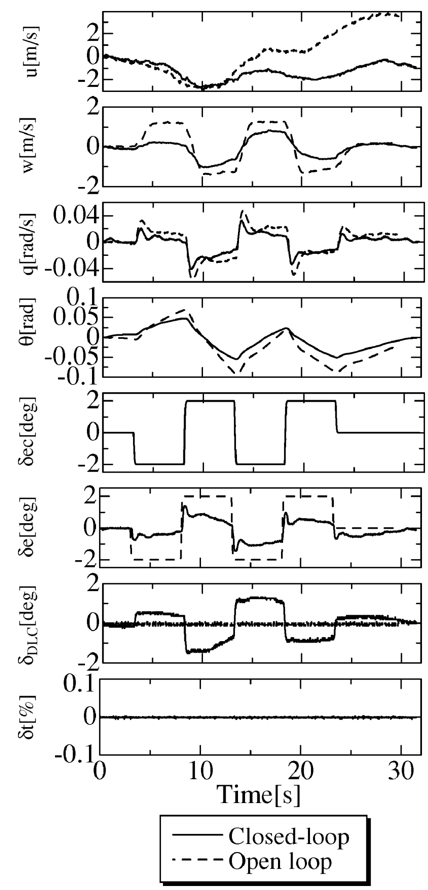

Fig. 8 Flight-test results.

Conclusions

A controller design method, which takes into account handling quality and robust stability, is proposed. The introduced parameter can evaluate the handling qualities of a high-order system, and it has a similar idea to an equivalent system in the treatment of high-frequency phase lag. Several time response features are adjusted using a quadratic performance index with a time-weighting function. Because the proposed method can adjust a number of response features directly, it is hoped that the time and cost of controller design will be reduced. Application to MuPAL-α (Multi-Purpose

Aviation Laboratory airplane) shows good handling qualities, robust stability subject to actuator uncertainties, and robustness for gust input.

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